Charles Darwin University

Final Examination

Family Name						
Given Name/s						
Student Number						
Teaching Period	Semester 2, 2017					

SMA211 – Mathematics 2B	DURATION	
	Reading Time:	10 minutes
	Writing Time:	180 minutes

INSTRUCTIONS TO CANDIDATES

1.1 This paper contains **six** questions. Answer all **Six (6)** questions.
1.2 All questions are of equal value, and parts carry marks as indicated.
1.3 All symbols, unless stated otherwise, have their usual meanings.
1.4 Read ALL questions carefully.
1.5 Answers without showing detailed working will attract little marks.

EXAM CONDITIONS

You may begin writing from the commencement of the examination session. The reading time indicated above is provided as a guide only.

This is a CLOSED BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

No dictionaries are permitted

ADDITIONAL AUTHORISED MATERIALS	EXAMINATION MATERIALS TO BE SUPPLIED
No additional printed material is permitted	1 x 20 Page Book 1 x Scrap Paper Formula Sheet/s

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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Question 1

(a) Find the Laplace transform of $f(t) = (a - bt)^2$, where *a* and *b* are constants. **(Marks 6)**

(b) Find the inverse Laplace transform of $F(s) = \dfrac{3s - 137}{s^2 + 2s + 401}$. **(Marks 7)**

(c) Find the Laplace transform of $f(t) = e^{2t} \sin t \cos t$. **(Marks 7)**

Question 2

(a) (i) Given that the Laplace transform of $y(t)$ is $\mathcal{L}(y(t)) = Y(s)$, show that the Laplace transform of its derivative = $\mathcal{L}(y'(t)) = sY(s) - y(0)$. **(Marks 4)**

(ii) Using the above results for the Laplace transform of a derivative in (i), find the Laplace transform of $y(t) = \sin^2(\pi t)$. **(Marks 4)**

(b) Find the inverse Laplace transform $\mathcal{L}^{-1}(F(s))$ of $F(s) = \dfrac{6(1 - e^{-\pi s})}{(s^2 + 9)}$. **(Marks 5)**

(c) Using the method of Laplace transformation solve the following initial value problem:
$y''(t) + y(t) = t$ if $0 < t < 1$ and 0 if $t > 1$, $y(0) = y'(0) = 0$. **(Marks 7)**

Question 3

(a) Find the response function $y(t)$ of the damped mass-spring system represented by the following ODE:
$y'' + 3y' + 2y = \delta(t - 1)$, with $y(0) = y'(0) = 0$. **(Marks 7)**

(b) Solve the following integral equation:

$$y(t) + \int_0^t (t - \tau) y(\tau) d\tau = 1 \quad .$$ **(Marks 6)**

(c) Find the solution of the initial value problem: $y'' + 5y' + 6y = u(t)$ with $y(0) = 0$, $y'(0) = 1$. **(Marks 7)**

Question 4

(a) Find the values of *a* and *b* for which the following function $u(x, y)$ is harmonic:

$$u(x, y) = ax^3 + bxy$$

and then find its harmonic conjugate $v(x, y)$. **(Marks 7)**

(b) (i) Find the path given in the following complex parametric form and sketch it.

$$z(t) = t + it^2 \ , \ (1 \le t \le 2).$$ **(Marks 3)**

(ii) Find the value of the following integral:

$$\int_C \sec^2 z dz \text{, from } z = \frac{\pi}{4} \text{ to } \frac{\pi}{4} i$$

along the path *C* given in (i). **(Marks 4)**

(c) Find the value of the following contour integral:

$$\oint_C \frac{dz}{z^2 + 4} dz$$

anticlockwise along the path $C: \ 4x^2 + (y-2)^2 = 4$. **(Marks 6)**

Question 5

(a) Using the ratio test find the centre and radius of convergence of the following power series:

$$\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2}(z-3i)^n \quad . \qquad \textbf{(Marks 5)}$$

(b) Find the first three terms of the Taylor series of $f(z)=\sin z$ with centre $z_0=\frac{\pi}{2}$.

(Marks 5)

(c) The efficiency [%] of seven Voith Francis turbines of runner diameter 2.4 m is measured as given below:

91.8 89.1 89.9 92.5 90.7 91.2 91.0

Represent these efficiencies by a box plot. **(Marks 5)**

(d) In a factory of producing screws, let the event A be "mean screw too slim" and event B be " "mean crew too short". Let $P(A)=0.1$ and the conditional probability that a slim screw is also too short be $P(B/A)=0.2$. Find the probability that a randomly picked up screw from the lot produced will be both too slim and too short. **(Marks 5)**

Question 6

(a) A lot contains 20 defective and 80 nondefective items. Two items are chosen at random, without replacement. What is the probability that both items are defective? **(Marks 5)**

(b) A pressure control apparatus contains 3 electronic tubes. The apparatus will not work unless all tubes are operative. If the probability of failure of each tube during some interval of time is 0.04, what is the corresponding probability of failure of the apparatus? **(Marks 5)**

(c) If the life of ball bearings has the density $f(x)=ke^{-0.2x}$ for $0\le x\le 10$ and 0 otherwise, what is the value of *k*? Find the probability $P(X\ge 5)$. **(Marks 5)**

(d) Find a 95% confidence interval for the percentage of cars on a certain highway that have poorly adjusted breaks, using a random sample of 500 cars stopped at a roadblock on that highway and 87 of which had poorly adjusted breaks. **(Marks 5)**